Derivation of "22 outs" (you can skip this)

It is straightforward to derive the "21 outs" formula when the player does has a pair, two pair or three of a kind. Here is the derivation.

Clearly if the player folds, then his EV is -2.

Because the player has a pair or better, for the dealer to beat the player, the dealer must also have at least a pair, and therefore will qualify. The limitation on the player's hand means that even if he wins, his Blind bet will push. Therefore if the player raises 1x and beats the dealer, then he wins 2 units. However, if the player raises 1x and loses to the dealer, then he loses 3 units.

Let N be the number of outs under consideration for the dealer to beat the player. Then the probability that the dealer's first card is an out is p = N/45. For his second card, the dealer most likely has an additional 3 "pair outs" to pair his first card and beat the player. He may also generate new straight or flush outs (call these one additional "out," so-called "runner-runner"). So, the probability of the dealer beating the player by hitting an out on his second card is approximately (N + 4)/44.

Overall, the probability that the dealer beats the player is then,

$$p = N/45 + [(45 - N)*(N + 4)]/(45*44)$$

Simplifying, we get:

$$p = (-N^2 + 85 N + 180)/(45*44)$$

Discounting the possibility of a tied hand, the EV for the player who raises 1x on the Turn/River bet is:

$$EV = p*(-3) + (1-p)*(2) = 2 - 5p.$$

We make the raise whenever EV > -2. That is, 2 - 5p > -2. Solving for p gives

$$p < 4/5.$$

That is, the player raises 1x when his chance of beating the dealer is 20% or higher.

Combining the two expressions for p, we see that EV > -2 whenever

$$(-N^2 + 85 N + 180)/(45*44) < 4/5.$$

Simplifying gives the quadratic equation,

$$N^2 - 85N + 1404 > 0$$

Using the quadratic formula gives roots:

- (1/2)*(85 + sqrt(1609)) = 62.6
- (1/2)*(85 - sqrt(1609)) = 22.4

For the quadratic equation to be positive, N must be either larger than both roots or smaller than both roots. That is, either N ≥ 63 or N ≤ 22. The first case is the "impossible solution," leading to the conclusion that there can be at most dealer 22 outs that can beat the player.